

**Example 2:** Find the third approximation  $x_3$  to the root of the equation  $x^3 - 2x - 7 = 0$  by using Newton's method with taking initial approximation  $x_1 = 2$ .

**Solution:**

Given function is

$$f(x) = x^3 - 2x - 7 = 0$$

So,  $f'(x) = 3x^2 - 2$

By Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - 2x_n - 7}{3x_n^2 - 2}$$

As given, the initial approximation,  $x_1 = 2$  then the second approximation is

$$x_2 = x_1 - \frac{x_1^3 - 2x_1 - 7}{3x_1^2 - 2} = 2 - \frac{8 - 4 - 7}{6 - 2} = 2 + \frac{3}{4} = 2.75$$

Then the third approximation is

$$x_3 = x_2 - \frac{x_2^3 - 2x_2 - 7}{3x_2^2 - 2} = 2.75 - \frac{20.7969 - 5.5 - 7}{15.125 - 2} = 2.1179$$

$$\Rightarrow x_3 = 2.1179$$

Thus, the third approximation to the root of given equation  $x^3 - 2x - 7 = 0$  is  $x_3 = 2.1179$ .